Episode 3 Extra Practice: Manual Limits and L'Hopital's Rule

Find the limits:

1. $\lim\limits_{x \to 3} \dfrac{2x^2 - 3x - 9}{x - 3}$

2. $\lim\limits_{x \to 0} \dfrac{\tan 5x}{9x}$

3. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{x^2}{\cos x}$

4. $\lim\limits_{h \to 0} \dfrac{\ln\; 5 + h\; - \ln\; 5}{h}$

5. $\lim\limits_{h \to 0} \dfrac{2\sqrt{x+4} - 2\sqrt{x}}{h}$

6. $\lim\limits_{h \to 0} \dfrac{\tanh - \tan 0}{h}$

Solutions:

1. $\lim_{x\to 3}\frac{(2x+3)(x-3)}{x-3} = \lim_{x\to 3}(2x+3) = 2\bullet 3+3 = 9$

2. $\lim_{x\to 0}\frac{1}{9}\bullet\frac{\sin 5x}{x\bullet\cos 5x} = \lim_{x\to 0}\frac{5}{9}\bullet\frac{\sin 5x}{5x}\bullet\frac{1}{\cos 5x} = \lim_{x\to 0}\frac{5}{9}\bullet 1\bullet\frac{1}{1} = \frac{5}{9}$

3. $\lim_{x\to\frac{\pi}{2}}\frac{2x}{-\sin x} = \frac{2\bullet\frac{\pi}{2}}{-\sin\frac{\pi}{2}} = \frac{\pi}{-1} = -\pi$

4. This is the derivative of the natural logarithm evaluated at x = 5. Since $\frac{d}{dx}\ln x = \frac{1}{x}$, and this derivative is being evaluated at x = 5, the solution is $\frac{1}{5}$

5. This is the derivative of $f(x) = 2\sqrt{x}$, so the solution is $f'(x) = \frac{1}{\sqrt{x}}$

6. This is the derivative of $f(x) = \tan x$ evaluated at x = 0

   So $f'(x) = \sec^2 x$

   And $f'(0) = \sec^2 0 = 1^2 = 1$