

## Episode 2 Extra Practice: Derivatives and Their Definitions

1. Find  $f'(x)$  using the definition of the derivative.

$$f(x) = x^3 - 2x^2 \qquad \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

2. Find  $f'(x)$  using the definition of the derivative.

$$f(x) = \sin x \qquad \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

**Solutions:**

1.  $f(x) = x^3 - 2x^2$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{[(x + \Delta x)^3 + 2(x + \Delta x)^2] - [x^3 - 2x^2]}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{[(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) - 2(x^2 + 2x\Delta x + (\Delta x)^2)] - [x^3 - 2x^2]}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2x^2 - 4x\Delta x - 2(\Delta x)^2 - x^3 + 2x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 4x\Delta x - 2(\Delta x)^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} [3x^2 + 3x\Delta x + (\Delta x)^2 - 4x - 2\Delta x]$$

$$\frac{dy}{dx} = 3x^2 + 3x \cdot 0 + (0)^2 - 4x - 2 \cdot 0$$

$$\frac{dy}{dx} = 3x^2 - 4x$$

2.  $f(x) = \sin x$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x}$$

Use identity:  $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{[\sin x \cdot \cos \Delta x + \cos x \sin \Delta x] - \sin x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{[\sin x \cdot \cos 0 + \cos x \sin \Delta x] - \sin x}{\Delta x}$$

Allow  $\Delta x$  to become 0 only in cos

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{(\sin x) \cdot 1 + \cos x \sin \Delta x - \sin x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\cos x \sin \Delta x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \cos x \cdot \frac{\sin \Delta x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} (\cos x) \cdot 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\frac{dy}{dx} = \cos x$$